

Calculation of volume change in ductile band structures

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Abstract—Volume changes incurred during the development of ideal band structures (infinite shear zones) can be readily calculated if one knows the principal directions and principal ratios of total strain within, and adjacent to, the ideal band structures. The calculation is greatly simplified if one knows the directions and ratios of the strain that generated the band structures, or if the total strain of the host rocks is zero. In some natural deformation zones, apparent volume losses according to the band model, exceed 30% and may be unreasonably high. Further determinations of apparent volume change will help to judge the physical viability of the band model and its strict geometrical requirements. For example, the propagation of embryonic deformation zones may lead to total strain patterns that differ greatly from those of band structures.

INTRODUCTION

SINCE 1971 the geometric model of ideal shear zones or band structures (Ramsay & Graham 1970, Cobbold 1977) has been widely used in structural geology and tectonics. Large amounts of strain data from natural deformation zones are becoming available, and these data may be used to judge the physical viability of the band model. For example, one might assess whether the total magnitudes of dilation predicted by the band model can actually be accommodated in continuous rock masses.

Ideal band structures are infinitely long and have parallel planar boundaries beyond which the total deformation is homogeneous. Within an ideal band structure, each concordant lamina experiences a finite translation in an arbitrary direction parallel to the boundaries, as well as a uniform change in thickness without concomitant longitudinal strain. The total heterogeneous deformation that generates the band structure may thus be simulated by multi-directional shear of a deck of porous cards under uniaxial compression normal to the deck. It must be remembered, however, that the band model relates a particular final state to its initial state irrespective of kinematic path.

Natural deformation zones that resemble band structures will deviate from these strict geometrical requirements. For example, the boundaries of major zones can be slightly oblique and non-planar. In addition, the deformation of the surrounding material can be slightly heterogeneous. The consequences of such minor departures from ideal band structures have not been assessed theoretically, so at what point the band model becomes inapplicable is not yet known.

It seems unreasonable on mechanical grounds that the total length of natural deformation zones should be generated instantaneously. More probably, they start as short sigmoidal embryos (Coward 1976) and proceed to grow to their final length. Due to superposition of biaxial on triaxial incremental strains, the fields of total strain

could depart significantly from those of ideal band structures.

No sigmoidal curvature is required if the embryonic deformation zones have characteristic triaxial strain fields in their end regions (Schwerdtner 1973, figs. 5 and 6, and Ramsay 1980, fig. 17b). Upon lateral propagation of the embryonic zones, the final geometric form of the middle segment of long zones could be indistinguishable from that of ideal band structures. Accordingly, k -values (Flinn 1962) of total band-generating strain (Schwerdtner, 1976) that differ significantly from unity would be interpreted as being indicative of huge volume changes (Grunsky *et al.*, 1980). The following contribution deals mainly with the problem of calculating such ostensible volume changes by assuming that natural deformation zones are ideal band structures and that the total deformation is perfectly continuous.

IDEAL BAND STRUCTURES

The perturbation of the deformation within ideal band structures, which have parallel planar boundaries and are surrounded by homogeneously strained material (Cobbold 1977), is given by

$$\begin{bmatrix} 1 & 0 & \partial U_1 / \partial X_3 \\ 0 & 1 & \partial U_2 / \partial X_3 \\ 0 & 0 & 1 + (\partial U_3 / \partial X_3)^{-1} \end{bmatrix} \quad (1)$$

where the reference axis X_3 is perpendicular to the boundaries and $\partial U_i / \partial X_j$ are the displacement gradients of the perturbed internal deformation. If the external deformation around the band structures is zero, a special case of homogeneous strain, then (1) describes the entire internal deformation (Cobbold 1977). At any point within ideal band structures, it is possible to select a reference frame such that the local perturbation of the deformation can be described by a matrix of biaxial deformation

$$\begin{bmatrix} 1 & 0 & dU_1/dX_3 \\ 0 & 1 & 0 \\ 0 & 0 & 1+(dU_3/dX_3) \end{bmatrix} \quad (2)$$

in which the axis X_1 is parallel to the direction of resultant simple shear and the axis X_3 normal to the boundaries.

As pointed out by Ramsay & Graham (1970), Cobbold (1977) and Schwerdtner (1977), the term $1+(dU_3/dX_3)$ in (2) represents a component of simple flattening (>1 for simple extension, <1 for simple contraction) which is solely responsible for the perturbation of the finite dilation. This dilation is positive in the case of simple extension and negative in the case of simple contraction (Cobbold 1977, Ramsay 1980). Ramsay & Graham (1970) and Ramsay (1980) chose to put $dU_3/dX_3 = \Delta$, the unit volume change or cubic dilation. This is correct numerically but obscures the fact that the perturbed displacement gradient also contains a component of longitudinal distortion (Schwerdtner 1977). As employed in their equations, Δ represents the total internal dilation only if the external deformation is volume conservative. For general states of homogeneous external deformation, a total internal dilation proves to be independent of the sequence of matrix superposition. Simple mathematical equations are derived in this paper which permit computation of local volume change or cubic dilation from principal ratios of total internal strain. Inverse values of these local dilation magnitudes may then be integrated along traverses across deformation zones to obtain the amount of overall normal extension or overall normal contraction. It will be assumed that natural features exist throughout ductile deformation zones which have recorded the total strain of the rock material.

ROTATIONAL DISTORTION AND DILATION

Consider a homogeneous linear transformation specified by an asymmetric matrix with the general coefficients

$$\begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \quad (3)$$

This finite deformation can be split into a state of rotational distortion with the components (\mathcal{D}_{ij}) and a state of dilation specified by the dilation factor $\delta = \sqrt[3]{(\Delta + 1)}$ (Schwerdtner, 1977). Accordingly,

$$D_{ij} = \delta \mathcal{D}_{ij} \quad (4)$$

For plane deformation (Jaeger 1962, p. 25)

$$\delta^3 = D_{11}D_{22} - D_{12}D_{21} \quad (5)$$

For triaxial deformation, δ^3 is equal to the expanded determinant of (3), which will be employed below. Equations 3–5 are of little direct use in estimating the dilation in narrow deformation zones because the structural geologist can determine merely the directions and principal ratios of internal and external strain. Most important, he rarely knows the direction and magnitude of

solid-body rotation (Schwerdtner 1979). If and how this limited information can be used to calculate the magnitude of dilation will now be explored.

BAND STRUCTURES IN UNSTRAINED MATERIAL

Consider band structures in rock bodies that have neither been distorted or compacted during lithification, nor subjected to subsequent tectonic deformation. The calculation of the total dilation at any point within such structures was treated previously (Schwerdtner 1977, p. T12). Given the principal ratios A/B , C/B of total biaxial strain (2) at a given location, where $A > B > C$, and $B = 1$, it was shown that the total dilation is

$$\Delta_t = ABC - 1 = \frac{A}{B} \times \frac{C}{B} - 1 \quad (6)$$

Note that (6) can be used for apparent band structures in prestrained material if natural features are available which have recorded the finite increment of deformation that generated the deformation zones. Such features could be ductile porphyroblasts or pseudomorphs which postdate the wall rock deformation but predate the apparent band structures. Excellent examples of incremental strain gauges within minor shear zones are found in prestrained amphibolites near Parry Sound, Grenville Province of Ontario (Schwerdtner *et al.* 1974).

In many regions, suitable incremental strain gauges are lacking, and the apparent volume change in deformation zones must be determined from the contrast between the internal and external total strains (Schwerdtner 1976). A treatment of this problem follows.

BAND STRUCTURES WITHIN STRAINED MATERIAL

It is customary to split the total internal deformation of band structures into two parts; an overall strain followed by a strain perturbation (Cobbold 1977). This procedure implies nothing about the kinematics of the deformation path. The perturbation strain may predate or postdate the overall strain, two possibilities which have been considered in recent papers by Cobbold (1977) and Ramsay (1980). More often, the two strains will be simultaneous or at least partly contemporaneous.

Consider a general overall strain that predates or postdates the strain increment which generates a band structure. Let both strains have the same axes of reference. Employing (5) or its triaxial equivalent for the total internal deformation, we find that the total internal dilation is

$$\Delta_t = \delta_b^3 \delta_e^3 - 1, \quad (7)$$

where δ_e is the dilation factor of the external strain or general overall strain and δ_b is the dilation factor of the strain increment which generates the band. Unlike the

total internal deformation (Ramsay 1980), Δ_i , does not depend on the sequence of superposition of the two finite strains. Because of the high symmetry of the dilation matrix (Schwerdtner 1977), this result is not unexpected. It demonstrates also that, given constant values of Δ_i and δ_e , the value of δ_b does not vary with the path of the internal deformation.

In most geological structures δ_e is unknown, but δ_b/δ_e may be estimated. If one puts $\delta_e = 1$ (no dilation of the host material), then $\Delta_i = \delta_b^3 - 1$ within the apparent band structures. This procedure will be followed in the remaining sections of the paper. Although the subsequent treatment assumes that band structures are superimposed on prestrained rocks, the results obtained are valid for any kinematic path.

Given natural features that indicate the direction of simple shear in apparent band structures, Δ_i can be readily calculated from the ratios of total strain. The situation is more difficult where the direction of resultant simple shear is unknown.

Shearing direction known

Ramsay (1980) recognized that the total internal deformation may be treated as two-dimensional strain if the X_1 -axis is parallel to the shearing direction. Using the classical transformation equations, he derived a series of implicit expressions that permit calculation of $\Delta_b = \Delta_i$. The following section contains an alternative treatment of the problem by means of coordinate geometry, which yields a set of explicit equations. The approach is similar to that adopted in a previous paper (Schwerdtner 1973).

Let the state of homogeneous prestrain in the X_1, X_3 plane of the band structure be represented by an ellipse with a specific orientation and relative length of the extreme radii A_1 and A_3 . This external hemi-ellipse is subjected to a deformation (2) within the X_1, X_3 plane to the internal hemi-ellipse of radii B_1 and B_3 (Fig. 1). The dilation Δ_i can be calculated without knowing the solid-body rotation associated with the external strain (cf. Ramsay 1980, equations 26–29).

Given a specific value of A_1/A_3 one may choose arbitrary magnitudes of A_1 and A_3 for the present purpose. The equation of an oblique ellipse is

$$\left\{ \frac{\cos^2 \alpha}{A_1^2} + \frac{\sin^2 \alpha}{A_3^2} \right\} X_1^2 - 2 \left\{ \frac{\sin \alpha \cos \alpha}{A_1^2} - \frac{\sin \alpha \cos \alpha}{A_3^2} \right\} \times X_1 X_3 + \left\{ \frac{\sin^2 \alpha}{A_1^2} + \frac{\cos^2 \alpha}{A_3^2} \right\} X_3^2 = 1 \quad (8)$$

where α is the angle between the direction of A_1 and the X_1 -axis. Call the bracketed sums in (8) R, S, T , respectively, and let γ be the resultant amount of simple shear and f the component of simple flattening in (2).

Now the deformation associated with the band structure is

$$\begin{aligned} x_1 &= X_1 + \gamma X_3 \\ x_3 &= f X_3 \end{aligned} \quad (9)$$

where x_1, x_3 are the coordinates of a displaced point. Prior to deformation, the coordinates of the same point are

$$\begin{aligned} X_1 &= x_1 - \gamma x_3 / f \\ X_3 &= x_3 / f. \end{aligned} \quad (10)$$

Substituting (10) into (8) gives the equation for the ellipse after deformation

$$x_1^2 R - 2 x_1 x_3 (R\gamma + S)/f + x_3^2 (R\gamma^2 + 2\gamma S + T)/f^2 = 1. \quad (11)$$

This corresponds to an equation

$$\left\{ \frac{\cos^2 \beta}{B_1^2} + \frac{\sin^2 \beta}{B_3^2} \right\} x_1^2 - 2 \left\{ \frac{\sin \beta \cos \beta}{B_1^2} - \frac{\sin \beta \cos \beta}{B_3^2} \right\} \times x_1 x_3 + \left\{ \frac{\sin^2 \beta}{B_1^2} + \frac{\cos^2 \beta}{B_3^2} \right\} x_3^2 = 1 \quad (12)$$

in which B_1, B_3 are the extreme radii of the total-strain ellipse (Fig. 1) and β is the angle between the direction of B_1 and the x_1 -axis. Equating the factors of x_1^2 in (11) and (12) gives

$$R = \frac{\cos^2 \beta}{B_1^2} + \frac{\sin^2 \beta}{B_3^2} = \frac{\cos^2 \beta}{B_1^2} + \frac{\rho^2 \sin^2 \beta}{B_1^2} \quad (13)$$

where $\rho = B_1/B_3$. This equation can be solved for B_1^2 , the only unknown quantity. Thus

$$B_1 = \sqrt{(\rho^2 \sin^2 \beta + \cos^2 \beta)/R}. \quad (14)$$

As $B_3 = B_1/\rho$ the area change within the X_1, X_3 plane can be calculated; this is equal to the local dilation through-out ideal band structures. It can be seen that

$$\Delta_i = (B_1 B_3 - A_1 A_3)/A_1 A_3. \quad (15)$$

Alternatively, $\Delta_i = f - 1$ can be found by equating the corresponding factors of $-2x_1 x_3$ and x_3^2 , respectively, in (11) and (12). This procedure also yields the value of γ and is similar to a method mentioned by Ramsay (1980, p. 96). The present method (13–15) is much simpler.

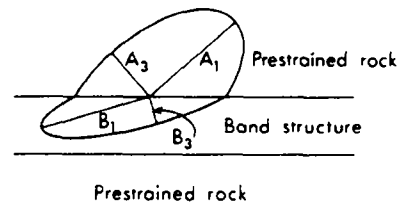


Fig. 1. Oblique section (X_1, X_3 plane) through the ellipsoids of external and internal total strain for the special case of uniform deformation in an ideal band structure whose direction of resultant simple shear is known.

Shearing direction unknown

Where natural indicators of the shearing direction are absent, one has to choose an arbitrary orientation of the X_1 -axis within the boundary plane of an apparent band structure. This means that an ellipse of prestrain in the X_1, X_3 plane transforms into another ellipse oblique to the

X_1, X_3 plane, an effect which renders the two-dimensional treatment invalid. Therefore (2) must be replaced by a matrix similar to (1). Instead of transforming an oblique ellipse, one must transform an oblique ellipsoid whose principal axes are $A_1 > A_2 > A_3$ and whose orientation is specified by a set of 9 direction cosines (a_{ij}). The corresponding equation is

$$\begin{aligned} a_{1j}^2 A_j^{-2} X_1^2 + a_{2j}^2 A_j^{-2} X_2^2 + a_{3j}^2 A_j^{-2} X_3^2 \\ + 2a_{1j} a_{2j} A_j^{-2} X_1 X_2 + 2a_{2j} a_{3j} A_j^{-2} X_2 X_3 \\ + 2a_{1j} a_{3j} A_j^{-2} X_1 X_3 = 1 \end{aligned} \quad (16a)$$

or more simply,

$$g_{ij} X_i X_j = 1 \quad (16b)$$

in which the $g_{ij} = g_{ji}$ are coefficients.

According to (1), the incremental deformation matrix at a point within a band structure is

$$\begin{bmatrix} 1 & 0 & S_{13} \\ 0 & 1 & S_{23} \\ 0 & 0 & f \end{bmatrix} \quad (17)$$

where S_{13} and S_{23} are components of simple shear and f is the component of simple flattening normal to the band structure. The transformation equations are therefore,

$$\begin{aligned} x_1 &= X_1 + S_{13} X_3 \\ x_2 &= X_2 + S_{23} X_3 \end{aligned} \quad (18a)$$

$$\begin{aligned} x_3 &= f X_3 \\ X_1 &= x_1 - S_{13} x_3 / f \\ X_2 &= x_2 - S_{23} x_3 / f \\ X_3 &= x_3 / f. \end{aligned} \quad (18b)$$

Combining (16) and (18),

$$\begin{aligned} g_{11} x_1^2 + g_{33} x_2^2 + f^{-2} (g_{11} S_{13}^2 + g_{22} S_{23}^2 + g_{33} \\ + 2g_{12} S_{13} S_{23} - 2g_{13} S_{13} - 2g_{23} S_{23}) x_3^2 \\ + 2g_{12} x_1 x_2 - 2f^{-1} (g_{22} S_{23} + g_{12} S_{13} - g_{23}) x_2 x_3 \\ - 2f^{-1} (g_{11} S_{13} + g_{12} S_{23} - g_{13}) x_1 x_3 = 1. \end{aligned} \quad (19)$$

The factors of x_i^2 , etc. may now be equated to those of the total strain ellipsoid. Its equation has the same form as (16), but the symbols B_i for the principal radii and b_{ij} for the direction cosines are employed. The factors of x_i^2 yield the equation

$$g_{11} = (b_{11}/B_1)^2 + (b_{12}/B_2)^2 + (b_{13}/B_3)^2 \quad (20)$$

in which b_{ij} is known, $B_1/B_2 = \rho$ and $B_1/B_3 = \sigma$. Thus

$$g_{11} = (b_{11}/B_1)^2 + (b_{12}\rho/B_1)^2 + (b_{13}\sigma/B_1)^2 \quad (21)$$

and more importantly,

$$B_1 = \{(b_{11}^2 + b_{12}^2 \rho^2 + b_{13}^2 \sigma^2)/g_{11}\}^{1/2}. \quad (22)$$

Using the values of ρ and σ one can calculate B_2 and B_3 . Finally, the volume change associated with the development of a band structure is

$$\Delta_t = (B_1 B_2 B_3 - A_1 A_2 A_3) / A_1 A_2 A_3. \quad (23)$$

The absolute values of A_i may be obtained by assuming zero volume change for the prestrain (cf. Schwerdtner 1977). It should be noted that, as in the method outlined by Ramsay (1980), the preceding equations are highly sensitive to small changes in principal directions (Schwerdtner 1974). In view of the information available to the geologist, this problem cannot be overcome at present.

Another problem is that the calculation of B_1 involves squares of ratios of principal strain, which can be quite inaccurate. According to Schnorr & Schwerdtner (1981), the precision of strain ratios obtainable with the Robin (1977) method is $\pm 10\%$ for a common type of deformed granite. This suggests that the equations for Δ_t derived above may be useless for identifying ostensible volume changes of a few tens of per cent. A calculation of the total error of Δ_t , however important, is beyond the scope of the present paper.

PRELIMINARY APPLICATIONS

Apparent volume loss in narrow deformation zones has been reported by Grunsky *et al.* (1980), Kligfield *et al.* (1981) and Stone & Schwerdtner (1981). The largest apparent values (Table 1) have been found within the Grenville Front Tectonic Zone (Grunsky *et al.* 1980). These values were calculated by applying (6) to strain data obtained by Themistocleous & Schwerdtner (1977) in a mylonite zone adjacent to which, a set of discordant dykes had suffered no appreciable strain.

Themistocleous & Schwerdtner's (1977) ratios of total internal strain are based on Ramsay's (1962) concept of flattening of concentric folds, which has been discredited by later modelling (Hudleston & Stephansson 1973). However, Ramsay's (1962) concept is possibly valid for the development of tight minor folds within major mylonite zones. In this structural environment, the intrinsic rheological contrast between a medium-grained tonalite and narrow aplitic dykes (cf. Hudleston 1973, p. 42) is apt to have decreased greatly as the buckles tightened and the original differences in grain size, texture and mineral assemblage between the two rock types diminished during progressive mylonitization (Themistocleous & Schwerdtner 1977).

Like Themistocleous & Schwerdtner (1977), Stone based his calculation of the strain tensor on measurements of magnetic anisotropy (Stone & Schwerdtner

Table 1. Local apparent volume loss ($-\Delta_t$) in the Grenville Front Tectonic Zone, Central Ontario

Fold No.	A/B	C/B	$-\Delta_t(\%)$	k (total strain)
1	3.06	0.17	48	0.50
2	6.34	0.06	62	0.43
3	3.19	0.10	68	0.33
4	1.49	0.22	67	0.19
6	1.61	0.32	49	0.35
7	1.71	0.21	64	0.26

1981). For a suite of mylonitized granitoid rocks, he was able to determine k (Flinn 1962) by using Fry's (1979) method and obtain close correspondence to k (magnetic anisotropy). The low values of k (total strain) in ductile deformation zones furnish the strongest evidence of apparent volume loss.

As pointed out by Themistocleous & Schwerdtner (1977), the errors in the strain ratios (and more drastically of Δ , in Table 1) could be quite large. In particular, the discordant aplitic dykes could have been thickened by 10–20% prior to mylonitization. This prestrain could have produced a flattening foliation in the tonalite which was later utilized as a plane of simple shear. Accordingly, the walls of the present mylonite zone would be parallel to the early flattening foliation (Schwerdtner 1973). This scenario may also be applicable to regional mylonite zones like the Sydney Lake fault zone (Schwerdtner *et al.* 1979) which transect elongate granitoid bodies that appear to have been emplaced concordantly.

The effect of concordant simple shear on the shape of oblate spheroids (primary flattening) was investigated using elementary matrix operations and several equations derived by Jaeger (1962, p. 23–33). The results are given in Table 2 and Fig. 2. It is apparent that the ostensible volume loss depends mostly on the magnitude of primary flattening, whereas the k -value of total strain depends on γ as well as the magnitude of primary flattening. Note that for reasonable magnitudes of indiscernible primary flattening (<20%) the ostensible volume loss is <30% within a realistic range of γ . A comparison of the corresponding magnitudes of total strain (Table 2) with those for Folds 4–7 (Table 1) shows that the apparent volume loss in the Grenville Front Tectonic Zone cannot be explained by the simple model of concordant shearing unless the value of $-\Delta$, has been systematically overestimated by >20%.

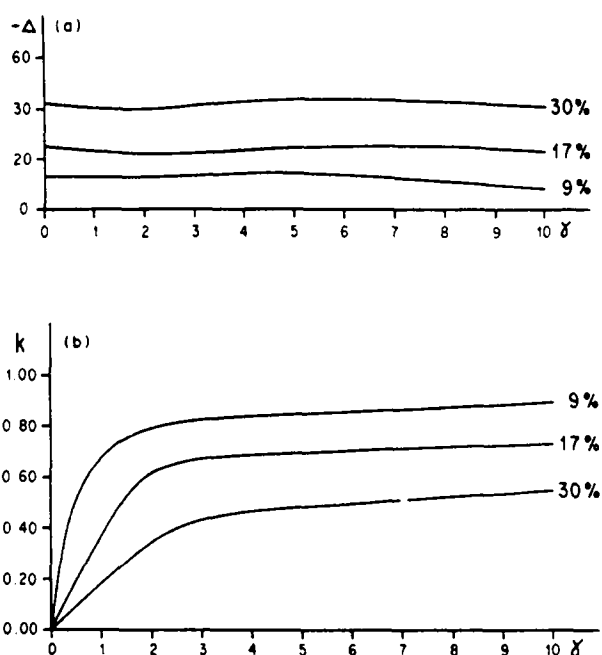


Fig. 2(a). Apparent volume loss ($-\Delta$) due to concordant simple shearing of three oblate spheroids whose degree of primary flattening is given in per cent (Table 2). (b) Change in k (Flinn 1962) with increasing γ and for different degrees of primary flattening (cf. Fig. 2a).

CONCLUSIONS

The apparent local dilation according to the band model is most readily calculated throughout narrow deformation zones if the host rocks are devoid of strain, or if geological features are available that recorded the deformation increment which generated the deformation zones.

Table 2. Effect of concordant simple shear on pure foliation with three primary flattening values

Amount (%) of primary flattening	Amount (%) of primary extension	γ (magnitude of simple shear)	k -value (total strain)	Principal magnitude of total strain			$-\Delta$ (ostensible volume decrease, %)
				A	B	C	
30	20	0	0	1.20	1.20	0.70	42
30	20	0.5	0.07	1.27	1.20	0.66	42
30	20	2	0.35	1.93	1.20	0.44	40
30	20	3	0.43	2.50	1.20	0.34	41
30	20	5	0.48	3.76	1.20	0.22	43
30	20	10	0.55	7.14	1.20	0.12	40
17	10	0	0	1.10	1.10	0.91	25
17	10	2	0.63	2.14	1.10	0.44	22
17	10	3	0.67	2.83	1.10	0.33	23
17	10	5	0.70	4.37	1.10	0.21	24
17	10	7.5	0.71	6.38	1.10	0.14	26
17	10	10	0.74	8.41	1.10	0.11	23
9	5	0	0	1.05	1.05	0.91	13
9	5	0.5	0.50	1.24	1.05	0.77	13
9	5	2	0.79	2.25	1.05	0.43	12
9	5	3	0.83	3.05	1.05	0.32	13
9	5	5	0.84	4.75	1.05	0.20	14
9	5	7.5	0.87	6.97	1.05	0.14	11
9	5	10	0.90	9.16	1.05	0.11	8

If these conditions do not apply it is nonetheless possible to calculate the apparent dilation (Δ_r) from the contrast between the external and internal total strains. Because Δ_r is independent of the kinematic path, the equations derived in this paper are not restricted to band structures that are superimposed on prestrained rocks. Most importantly, the equations permit the calculation of Δ_r independently of γ and differ in form from those of Ramsay (1980, p. 96).

Some of the natural deformation zones studied to date appear to have suffered large volume losses. It is unlikely that this apparent dilation is due to concordant primary fabrics in the original rocks. Apparent volume losses of > 30% may be unrealistic for some rock types, and may indicate that the band model is inadequate for deformation zones in such rock bodies.

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REFERENCES

- Cobbold, P. R. 1977. Description and origin of banded deformation structures. I. Regional strain, local perturbations, and deformation bands. *Can. J. Earth Sci.* 14, 1721–1731.
- Coward, M. P. 1976. Strain within ductile shear zones. *Tectonophysics* 34, 181–197.
- Flinn, D. 1962. On folding during three-dimensional progressive deformation. *Q. Jl geol. Soc. Lond.* 118, 385–433.
- Fry, N. 1979. Random point distributions and strain measurement in rocks. *Tectonophysics* 60, 89–105.
- Grunsky, E. C., Robin, P. Y. F. & Schwerdtner, W. M. 1980. Orientation of feldspar porphyroclasts in mylonite samples from the southern Churchill Province, Canadian Shield. *Tectonophysics* 66, 213–224.
- Jaeger, J. C. 1962. *Elasticity, Fracture and Flow*. Methuen and Co., London.
- Hudleston, P. J. 1973. Fold morphology and some geometrical implications of theories of fold development. *Tectonophysics* 16, 1–46.
- Hudleston, P. J. & Stephansson, O. 1973. Layer shortening and fold-shape development in the buckling of single layers. *Tectonophysics* 17, 299–321.
- Kligfield, R., Carmignani, L. & Owens, W. H. 1981. The strain field of a ductile shear zone in the northern Apennines (Abstract). *J. Struct. Geol.* 3, 188–189.
- Ramsay, J. G. 1962. The geometry and mechanics of similar type folds. *J. Geol.* 70, 309–327.
- Ramsay, J. G. 1980. Shear zone geometry: a review. *J. Struct. Geol.* 2, 83–99.
- Ramsay, J. G. & Graham, R. H. 1970. Strain variation in shear belts. *Can. J. Earth Sci.* 7, 786–813.
- Robin, P. Y. F. 1977. Determination of geologic strain using randomly oriented strain markers of any shape. *Tectonophysics* 42, T7–T16.
- Schwerdtner, W. M. 1973. Schistosity and penetrative mineral lineation as indicators of paleostrain directions. *Can. J. Earth Sci.* 10, 1233–1243.
- Schwerdtner, W. M. 1974. Erratum: schistosity and penetrative mineral lineation as indicators of paleostrain directions. *Can. J. Earth Sci.* 11, 210.
- Schwerdtner, W. M. 1976. A problem of nomenclature in paleostrain analysis. *Tectonophysics* 30, T1–T2.
- Schwerdtner, W. M., 1977. Distortion and dilation in paleostrain analysis. *Tectonophysics* 40, T9–T13.
- Schwerdtner, W. M. 1979. Natural indicators of solid-body rotation in deformed rocks. *Tectonophysics* 53, T15–T20.
- Schwerdtner, W. M., Waddington, D. H. & Stollery, G. 1974. Polycrystalline pseudomorphs as natural gauges of incremental paleostrain. *Neues Jb. Miner. Mh.* 3/4, 174–182.
- Schwerdtner, W. M., Stone, D., Osadetz, K., Morgan, J. & Stott, G. M. 1979. Granitoid complexes and the Archean tectonic record in the Southern part of northwestern Ontario. *Can. J. Earth Sci.* 16, 1965–1977.
- Schnorr, P. & Schwerdtner, W. M. 1981. A test for sample size and precision of Robin's method of strain analysis. *Tectonophysics* 73, T1–T8.
- Stone, D. & Schwerdtner, W. M. 1981. Total strain within a major mylonite zone, southern Canadian shield. (Abstract). *J. Struct. Geol.* 3, 193–194.
- Themistocleous, S. G. & Schwerdtner, W. M. 1977. Estimates of distortional strain in mylonites from the Grenville Front Tectonic Zone, Tomiko area, Ontario. *Can. J. Earth Sci.* 14, 1708–1720.